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LEAKAGE CALCULATION THROUGH CLEARANCES

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ABSTRACT

Gas leakage flow through small clearances is modeled using two-dimensional (2-D) and quasi one-dimensional (1-D) flow analysis. For 2-D approach, a dimensionless form of Navier-Stokes equations for two-dimensional, laminar, viscous and compressible fluids are employed to compute the detailed flow field across the clearance. The 2-D study also establishes that when clearance (δ) is very small compared to the length (L), i.e., $\delta/L < 0.01$, a quasi 1-D model is adequate to predict the leakage. The quasi 1-D model includes viscous, convective and dissipative effects. An integral method is used to derive the relationship between mass flow rate and other parameters. It is found that both convective and viscous effects are equally important for leakage flow. Therefore, the solution of the quasi 1-D model can be used efficiently in performance analysis of compressors including such gas leakage.

INTRODUCTION

Losses due to leakage through tip and flank clearances of scroll compressor are among the most important factors for degrading performance. In design of scroll (or any other types of) compressors, great care is taken to avoid the excessive leakage loss. Although precision tools today allow that contact surfaces be machined within a few microns, it is still in demand to fully understand how much leakage would result under specific conditions in order to estimate losses and conduct performance analysis.

Various models have been used before to evaluate leakage flow through clearances. One is based on 1-D inviscid nozzle flow (for example, see [1]). The deficiency of 1-D inviscid model is that viscous effects, while important, are neglected. On the other hand, purely 1-D viscous flow (Stokes flow) ignores the convection effect, which may be significant when pressure gradient is large [2]. In addition, other quasi 1-D models were also employed to calculate the leakage [3].

To fully understand the flow through clearances, an approach based on computational fluid dynamics (CFD) is used in the present project. It is aimed at calculating the detailed flow field, including velocity, pressure and mass flow across the clearance and assessing the threshold as to when a quasi 1-D model is sufficient enough to predict the mass flow. The setup of the problem is illustrated in Fig. 1.

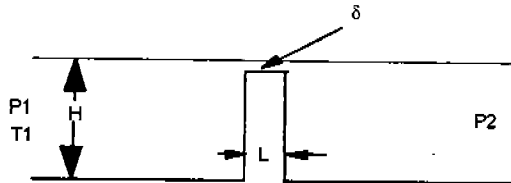


Fig. 1 Tip Clearance Geometry.

One-dimensional models or quasi 1-D models can be a good approximation only when the ratio δ/L is very small. The present study begins with 2-D CFD approach and is followed by a quasi 1-D model when δ/L is small, which is usually the case for tip or flank clearances of scroll compressors.

2-D APPROACH

Two-dimensional Navier-Stokes equations were used to calculate the flow field. The flow was assumed to be compressible, viscous and laminar. Ideal gas law was employed for equation of state. The system of partial differential equations governing this type of flow is described as follows in non-dimensional form for Cartesian coordinate:

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = \frac{1}{Re} \left(\frac{\partial E_v}{\partial x} + \frac{\partial F_v}{\partial y} \right) \quad (1)$$

$$p = (\gamma - 1) \left[e - \frac{1}{2} \rho (u^2 + v^2) \right] \quad (2)$$

$$a^2 = \gamma p / \rho \quad (3)$$

where $Q = [\rho, \rho u, \rho v, e]^T$, $E = [\rho u, \rho u^2 + p, \rho uv, u(e + p)]^T$, $F = [\rho v, \rho uv, \rho v^2 + p, v(e + p)]^T$, ρ is density, u and v are velocity components. e is total energy, p is pressure, a is the speed of sound and γ is the ratio of specific heats. Reynolds number Re is defined as $Re = \rho_j a_j H / \mu_j$. The viscous terms are:

$$E_v = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ u\tau_{xx} + v\tau_{xy} + \mu Pr^{-1}(\gamma - 1)^{-1} \frac{\partial a^2}{\partial x} \end{bmatrix} \quad F_v = \begin{bmatrix} 0 \\ \tau_{yx} \\ \tau_{yy} \\ u\tau_{yx} + v\tau_{yy} + \mu Pr^{-1}(\gamma - 1)^{-1} \frac{\partial a^2}{\partial y} \end{bmatrix} \quad (4)$$

where shear stresses $\tau_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$, etc. and Pr is Prandtl number. The viscosity follows Sutherland's law:

$$\frac{\mu}{\mu_\infty} = \left(\frac{T}{T_\infty} \right)^{3/2} \frac{T_\infty + 110}{T + 110} \quad (5)$$

A finite-volume method [4] was used to discretize Eq. (1) so that a system of algebraic equations for five unknown variables (ρ , u , v , p , e) were derived. With the appropriate upstream boundary conditions where total pressure and temperature were specified and downstream conditions where static pressure was set, the equations were integrated to steady state using MacCormack method; which is second-order accurate. Upstream and downstream locations were defined as some distance up and down the step (about 10 times of channel height H).

QUASI 1-D APPROACH

The underlying assumption of quasi 1-D steady state model is that the ratio of clearance height δ to the length of the clearance L is very small: $\delta/L \ll 1$. As established in the previous section, this ratio should generally be less than 0.01 for validity and accuracy of quasi 1-D model. The schematic geometry of quasi 1-D model is shown in Fig. 2.

Based on the basic geometric assumption, the pressure, temperature and density of the gas vary in the x -coordinate:

$$p = p(x), \quad T = T(x), \quad \rho = \rho(x) \quad (6)$$

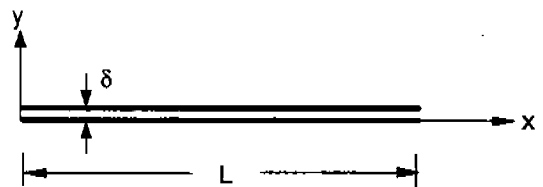


Fig. 2 Geometry for Quasi 1-D Model.

As a result, the governing equations, Eq. (1)-(2), degenerate to a set of boundary-layer-like equations:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (7)$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{dp}{dx} + \mu \frac{\partial^2 u}{\partial y^2} \quad (8)$$

$$\rho C_p \left(u \frac{dT}{dx} \right) = u \frac{dp}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (9)$$

$$p = \rho RT \quad (10)$$

The above set of equations were obtained by eliminated those terms in 2-D equations which are either zero due to the assumption (Eq. (6)) or very small due to the fact that $\delta/L \ll 1$. However, the dominating effects for small clearances were taken into account: viscosity and convection. Also included in the energy equation (9) was the viscous dissipation term which is significant when high speed and larger viscosity are present. It is noted that if this dissipation term was neglected, the energy equation (9), together with the equation of state (10), reduces to familiar adiabatic relationship for an adiabatic wall:

$$p / \rho^\gamma = \text{Const} \quad (11)$$

The set of equations (7)-(10) is also applicable to flank clearance case if curvature is large compared to the length of clearance passage, i.e., $L/R \ll 1$ (see Fig. 3). In this case, the clearance height δ is a function of streamwise coordinate and x coordinate is replaced by streamwise coordinate, s .

The solution of the equations (7)-(10) was obtained via an integral method. The mass flow rate was defined as:

$$\dot{m} = \int_0^\delta \rho u dy$$

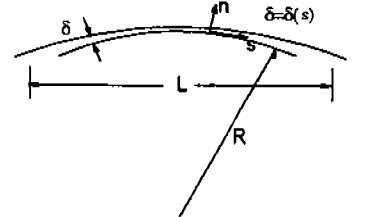


Fig. 3 Flank Clearance Geometry.

(12)

After lengthy derivation, the following equation defining relationship between mass flow rate and pressure, state variables, viscosity and geometry was obtained:

$$\frac{\delta^4 (\gamma - 1) \mu}{80 \gamma p} G^3 + \frac{1}{120} \left(\delta^3 \frac{d\delta}{dx} + \frac{\delta^4}{\gamma p} \frac{dp}{dx} \right) G^2 + \frac{\mu}{\rho} G - \frac{1}{\rho} \frac{dp}{dx} = 0 \quad (13)$$

$$\dot{m} = -\frac{\rho \delta^3}{12} G$$

The cubic term in Eq. (13) is identified as the contribution from viscous dissipation in the energy equation; the quadratic term represents convection effect in the momentum equation, and the linear term is due to viscosity. The equation (13) is applicable for both tip and flank clearance, as long as the curvature of the flank is much larger than the effective passage length.

RESULTS AND DISCUSSION

For 2-D calculation, the width of the tip was set to be such that $L/H = 0.0726$. The clearance height δ/H was allowed to vary to see the 2-D effect. In the present study, the flow is in the laminar regime for Reynolds number (Re) ranging from 500 to 1500.

Figure 4 shows that the mass flow rate decreases significantly with the reduction of clearance height δ for fixed pressure ratio and Reynolds number. For these values of δ/H , two-dimensional flow persists. In Fig. 5, the variation of mass flow rate with Reynolds number is shown for $\delta/H = 0.05$. It is observed that at higher Reynolds number, namely, when the gas is less viscous, the increase of pressure ratio leads to flatter portion of the curve, indicating the restrictive effect of compressible flow through a small opening.

It was shown from the 2-D computation that pressure ratio (inviscid effect) and Reynolds number (viscous effect) play important roles. The pressure drop was found to occur mostly across the step. For the smaller clearance case when $\delta/H = 0.025$, a quasi-linear drop across the clearance of pressure was observed (Fig. 6), suggesting that continuously

decreasing clearance height would lead to quasi 1-D flow within the clearance. The flow was stagnant in most regions away from the step for this case.

To demonstrate the result of 2-D calculation in absolute units, the mass flow rate is plotted against the clearance in Fig. 7, for a pressure ratio of $P_2/P_1=0.85$. Also shown in Fig. 7 is the result of quasi 1-D model and its discrepancy with the 2-D result. It is important to note that when the clearance height decreases, the two results converge. It is seen that if $\delta/L < 0.01$, the error between 2-D and quasi 1-D calculations is within a few percent, which is very acceptable for engineering calculations.

Figure 8 shows the results using present quasi 1-D model with those obtained by Suefuji et al. [3], who employed a set of ordinary differential equations and estimated viscous effect through friction factor. It was found that the agreement with Suefuji et al. [3] is good considering that Suefuji et al. [3] did not provide all the data needed for the present calculation (temperature and viscosity).

In Fig. 9, the mass flow rate is plotted against pressure ratio for several tip clearance values. The clearance length was set at 3 mm and its width was standardized at 1 inch. It can be observed that when the clearance is large (>20 microns) the reduction of pressure ratio does not increase the mass flow rate. This is an indication that the strong convection effect due to pressure ratio results in choked flow. The Mach number was indeed found to be close to unity at the inlet for these cases.

Flank clearance leakage was simulated and the results are presented in Fig. 10. A nozzle-like passage of flank clearance allowed a slightly easier flow through the passage. This is demonstrated by comparing the mass flow rate with those shown in Fig. 9 for the same clearance. At 20 microns, flank leakage is not yet significantly choked.

The influence of various effects including viscosity, convection and dissipation is shown in Fig. 11 and Fig. 12 for two tip clearance values. At $\delta=10$ microns, the omission of dissipation and especially convection gives significant error. Thus, purely viscous model is not adequate for predicting leakage. At $\delta=5$ microns, the discrepancy decreases as viscosity becomes more important. It is also observed that the dissipation is not as important as convection and viscosity.

SUMMARY

Two analytical approaches were pursued in study of leakage flow. Through small clearances the 2-D flow analysis using Navier-Stokes equations were found to be more general and applicable to larger clearances. However, the computational cost is relatively high compared to simplified quasi 1-D modeling. In addition, the 2-D model, presently, is limited to laminar and regular geometry. Further research is necessary to extend the 2-D approach to turbulent flow in an irregular geometry. Nevertheless, the 2-D modeling helped establish the threshold when quasi 1-D analysis is adequate. It was found that $\delta/L < 0.01$ is a necessary condition before applying 1-D model, which is almost always the case in practical design of compressors.

For quasi 1-D modeling, important factors such as viscosity and convection were considered in the model for leakage flow. Flank leakage as well as tip leakage were modeled under certain conditions. It was found that neither effect can be neglected for a 10-micron clearance when the length is about 3 mm. Also the choked effect of the leakage flow was observed when the pressure difference increased for larger clearances (20 to 40 microns). The present 1-D result was compared to Suefuji et al. [3] and general agreement was observed.

ACKNOWLEDGMENTS

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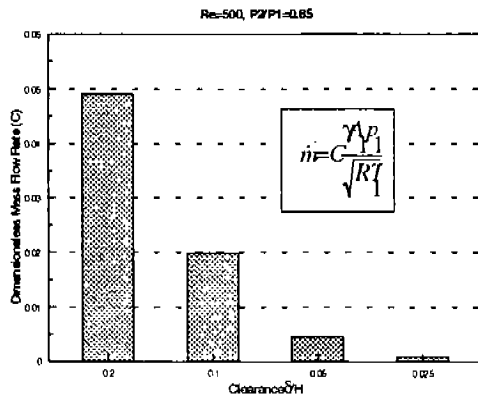


Fig. 4 Mass Flow Rate vs. Clearances. 2-D results

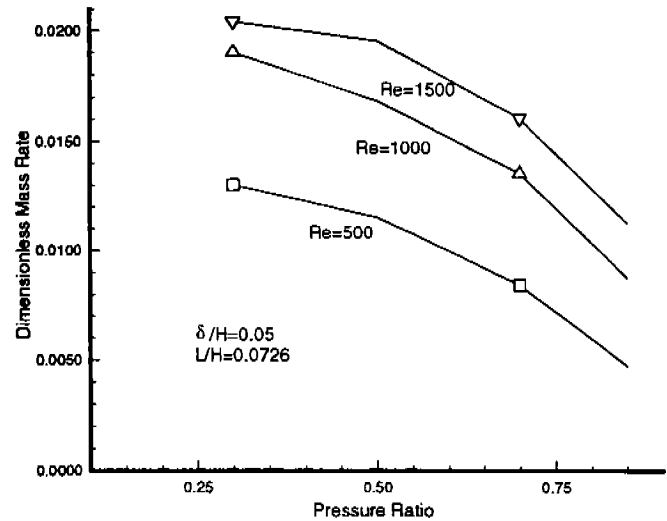


Fig. 5 2-D Mass Rate Through Clearance

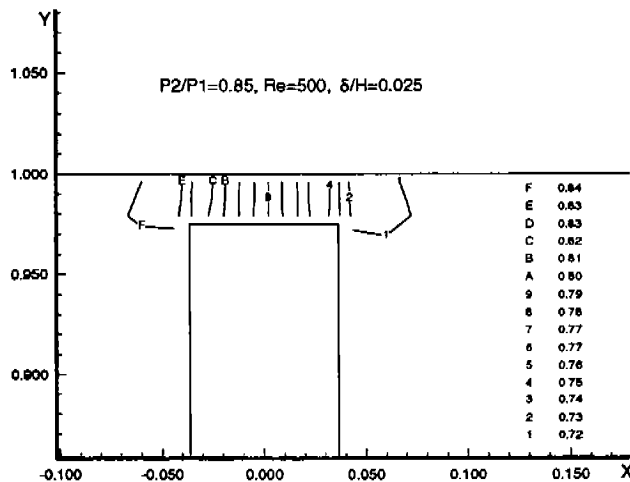


Fig. 6 Pressure Contour across Tip Clearance (2-D Result)

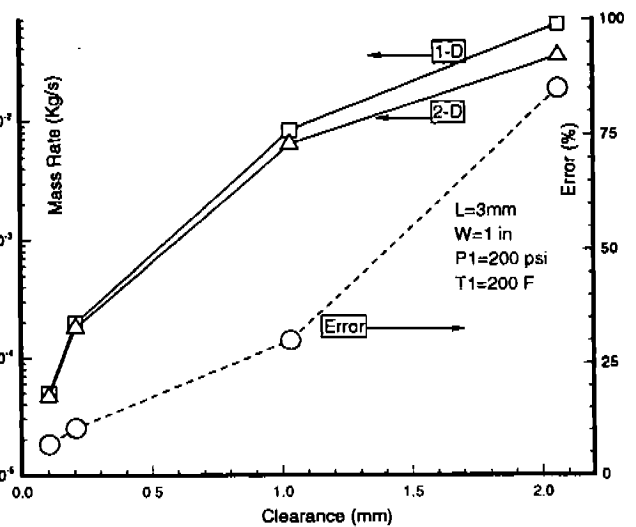


Fig. 7 Leakage Flow: 2-D vs. 1-D

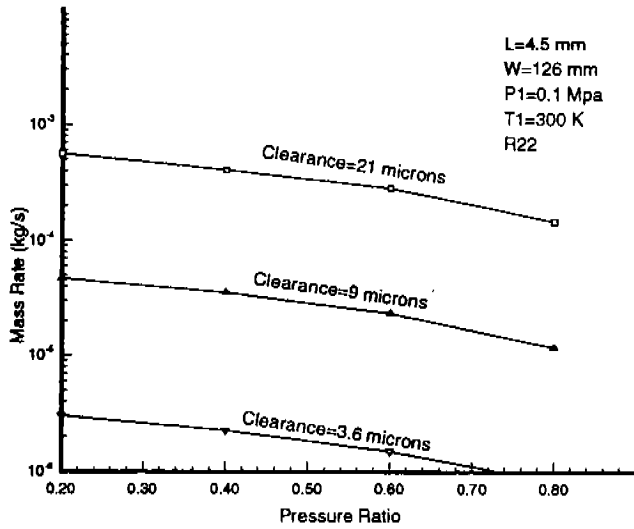


Fig. 8 Tip Leakage Using Data of Suefui et al. [3].

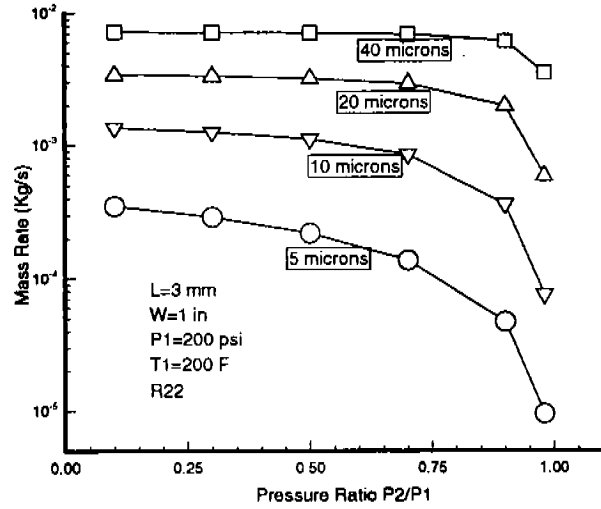


Fig. 9 Leakage Through Tip Clearance

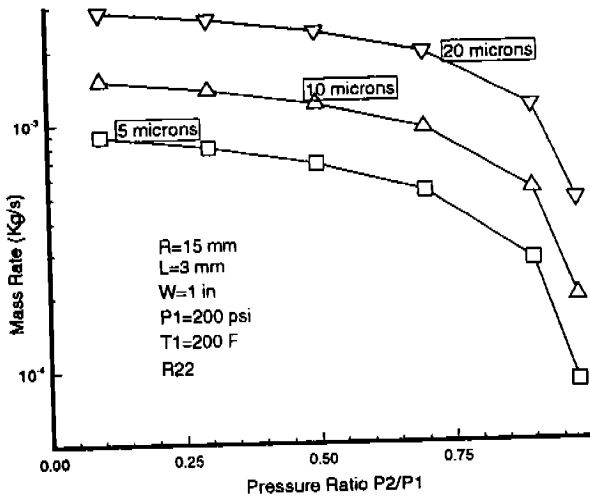


Fig. 10 Leakage Through Flank Clearance

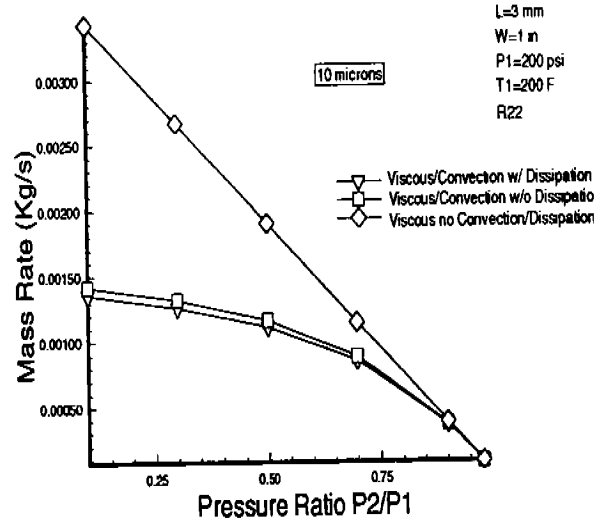


Fig. 11 Effect of Viscosity, Convection and Dissipation on Leakage. $\delta=10 \text{ microns}$.

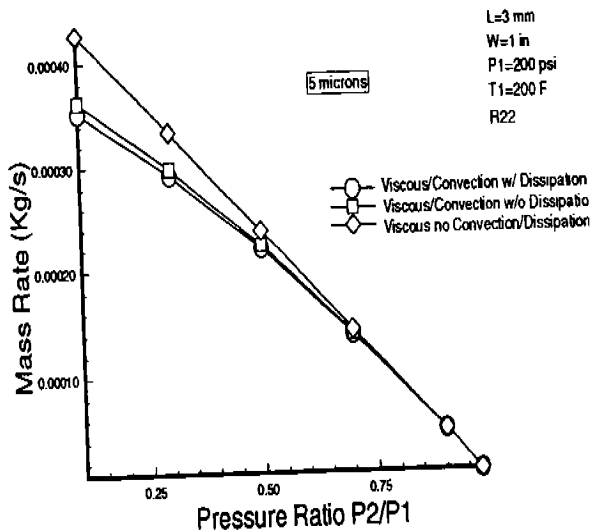


Fig. 12 Effect of Viscosity, Convection and Dissipation on Leakage. $\delta=5 \text{ microns}$.